Lazy Abstraction with Interpolants for Arrays

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Context

Software model checking:

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- Transition-relation representation of input program
- Predicate abstraction [GS97]
- Lazy Abstraction [HJMS02]
  - different degrees of precision for different parts of the program

Several abstraction refinement strategies

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Interpolants from (unsatisfiable) formulas representing infeasible counterexamples [HJMM04, McM06]

1. $\phi = \phi_1 \wedge \cdots \wedge \phi_n$ is satisfiable iff $\pi = \tau_1 \cdots \tau_n$ is feasible
2. Retrieve a set of (quantifier-free) formulas $\{\psi_0, \ldots, \psi_n\}$ s.t.
   - $\psi_0 \equiv \top$
   - $\psi_n \equiv \bot$
   - $\psi_{i-1} \wedge \phi_i \models \psi_i$
   - ψ_i is over the common signature of ϕ_i and ϕ_{i+1}

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$$\forall i, j.(0 \leq i < j < a.length) \Rightarrow a[i] \leq a[j]$$

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$$\forall i, j.(0 \leq i < j < a.length) \Rightarrow a[i] \leq a[j]$$

2. No quantifier-free interpolation for the "standard" theory of array [KMZ06]

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⇒ Redefine the interpolation-based Lazy Abstraction approach:
  1. Quantifier handling and Array-reasoning adapted from MCMT
  2. New quantifier-free interpolation algorithm for arrays

Outline

1 Array-based Transition Systems

2 Unwinding Array-based Transition Systems

3 Refinement with Interpolants

4 Completeness

5 Experiments

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Array-based Transition Systems

Class of guarded assignment systems with array state variables

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- A mono-sorted (`INDEX`) theory $T_I = (\Sigma_I, \mathcal{C}_I)$ for indexes of arrays
- A multi-sorted ($\mathtt{ELEM}_l$) theory $T_E = (\Sigma_E, \mathcal{C}_E)$ for data inside arrays

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- A theory $A_I^E = (\Sigma, \mathcal{C})$ linking T_I and T_E

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- A theory $A_I^E = (\Sigma, \mathcal{C})$ linking T_I and T_E
  - sort symbols of A_I^E are $\mathtt{INDEX}, \mathtt{ELEM}_l$ and $\mathtt{ARRAY}_l$
  - $\Sigma = \Sigma_I \cup \Sigma_E \cup \{\_[\_]_l\}_l$
    - $\_[\_]_i : \mathtt{ARRAY}_i \times \mathtt{INDEX} \rightarrow \mathtt{ELEM}_i$

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  - $\Sigma = \Sigma_I \cup \Sigma_E \cup \{\_[\_]_l\}_l$
    - $\_[\_]_i : \texttt{ARRAY}_i \times \texttt{INDEX} \to \texttt{ELEM}_i$
  - $\mathcal{M} \in \mathcal{C}$ iff
    - $\texttt{ARRAY}_l^{\mathcal{M}} = [\texttt{INDEX}^{\mathcal{M}} \to \texttt{ELEM}_l^{\mathcal{M}}]$
    - $\_[\_]_l^{\mathcal{M}}$ is function application
    - $\mathcal{M}|_{\Sigma_I} \in \mathcal{C}_I$ and $\mathcal{M}|_{\Sigma_E} \in \mathcal{C}_E$

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$\mathbf{v} = \{\mathbf{a}, \mathbf{c}, \mathbf{d}\}$

- $\mathbf{a}$ is a set of variables of sort $\mathtt{ARRAY}_l$
- $\mathbf{c}$ is a set of variables of sort $\mathtt{INDEX}$
- $\mathbf{d}$ is a set of variables of sort $\mathtt{ELEM}_l$

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- $\texttt{pc} \in \mathbf{d}$ ranging over a set L of locations
- $I(\mathbf{v}) \triangleq (\texttt{pc} = l_I)$ is the initial state of $\mathcal{S}$
- $U(\mathbf{v}) \triangleq (\texttt{pc} = l_E)$ is the error state of $\mathcal{S}$

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The τ_h's are guarded assignments in functional form

$$\exists \underline{k} \begin{pmatrix} \phi(\underline{k}, \mathbf{a}[\underline{k}], \mathbf{c}, \mathbf{d}) \wedge \\ \mathbf{a}' = \lambda j.\, G(\underline{k}, \mathbf{a}[\underline{k}], \mathbf{c}, \mathbf{d}, j, \mathbf{a}[j]) \wedge \\ \mathbf{c}' = H(\underline{k}, \mathbf{a}[\underline{k}], \mathbf{c}, \mathbf{d}) \wedge \\ \mathbf{d}' = K(\underline{k}, \mathbf{a}[\underline{k}], \mathbf{c}, \mathbf{d}) \end{pmatrix}$$

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For all τ_h:

- $\phi(\underline{k}, \mathbf{a}[\underline{k}], \mathbf{c}, \mathbf{d}) \models \mathtt{pc} = l$; $\quad src(\tau_h) = l$
- $\mathtt{pc}' = l'$; $\quad trg(\tau_h) = l'$

Array-based Transition Systems

Translation from source code

```
function find ( int a[ ] , int n ) {
    c = 0;
    while (c < a.length ∧ a[c] ≠ n) c = c + 1;
    if ( c ≥ a.length ∧ ∃x.(x ≥ 0 ∧ x < a.length ∧ a[x] = n) )
        ERROR;
}
```

```
  function find ( int a[ ] , int n ) {
1     c = 0;
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3     if ( c ≥ a.length ∧ ∃x.(x ≥ 0 ∧ x < a.length ∧ a[x] = n) )
4        ERROR;
  }
```

$T_I = \mathcal{LIA}$ with a constant $\mathtt{a.length}$

$T_E = \mathcal{LIA}$ with a constant $\mathtt{n}$ $\cup$ $\{1, 2, 3, 4\}$

$\mathbf{a} = \{\mathtt{a}\}$, $\mathbf{c} = \{\mathtt{c}\}$, $\mathbf{d} = \{\mathtt{pc}\}$

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Translation from source code

function $\texttt{find}$ ($\texttt{int a}[\,]$, $\texttt{int n}$) {

1 $\quad \texttt{c} = 0;$

2 $\quad \texttt{while}\ (\texttt{c} < \texttt{a.length} \wedge \texttt{a}[\texttt{c}] \neq \texttt{n})\ \texttt{c} = \texttt{c} + 1;$

3 $\quad \texttt{if}\ (\ \texttt{c} \geq \texttt{a.length} \wedge \exists x.(x \geq 0 \wedge x < \texttt{a.length} \wedge \texttt{a}[x] = \texttt{n})\ )$

4 $\qquad \texttt{ERROR};$

}

$$l_I = 1 \qquad\qquad l_E = 4$$

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    function find ( int a[ ] , int n ) {
1       c = 0;
2       while (c < a.length ∧ a[c] ≠ n) c = c + 1;
3       if ( c ≥ a.length ∧ ∃x.(x ≥ 0 ∧ x < a.length ∧ a[x] = n) )
4           ERROR;
    }
```

$$\tau_1 \equiv \mathtt{pc} = 1 \wedge \mathtt{pc}' = 2 \wedge \mathtt{c}' = 0$$

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  function find ( int a[ ] , int n ) {
1    c = 0;
2    while (c < a.length ∧ a[c] ≠ n) c = c + 1;
3    if ( c ≥ a.length ∧ ∃x.(x ≥ 0 ∧ x < a.length ∧ a[x] = n) )
4       ERROR;
  }
```

$$\tau_2 \equiv \quad \mathtt{pc} = 2 \wedge \mathtt{c} < \mathtt{a.length} \wedge \quad \mathtt{a}[\mathtt{c}] \neq \mathtt{n} \quad \wedge\, \mathtt{pc}' = 2 \wedge \mathtt{c}' = \mathtt{c} + 1$$

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```

$$
\begin{aligned}
\tau_2 &\equiv \quad \mathtt{pc} = 2 \wedge \mathtt{c} < \mathtt{a.length} \wedge \quad \mathtt{a}[\mathtt{c}] \neq \mathtt{n} \quad \wedge \mathtt{pc}' = 2 \wedge \mathtt{c}' = \mathtt{c} + 1 \\
&\equiv \exists x. \quad \mathtt{pc} = 2 \wedge \mathtt{c} < \mathtt{a.length} \wedge \quad x = \mathtt{c} \wedge \mathtt{a}[x] \neq \mathtt{n} \quad \wedge \mathtt{pc}' = 2 \wedge \mathtt{c}' = \mathtt{c} + 1
\end{aligned}
$$

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Translation from source code

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  function find ( int a[ ] , int n ) {
1   c = 0;
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3   if ( c ≥ a.length ∧ ∃x.(x ≥ 0 ∧ x < a.length ∧ a[x] = n) )
4     ERROR;
  }
```

$$\tau_3 \equiv \mathtt{pc} = 2 \land \mathtt{c} \geq \mathtt{a.length} \land \mathtt{pc}' = 3$$

```
   function find ( int a[ ] , int n ) {
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4         ERROR;
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```

$$\tau_4 \equiv \exists x.\mathtt{pc} = 2 \wedge x = \mathtt{c} \wedge \mathtt{a}[x] = \mathtt{n} \wedge \mathtt{pc}' = 3$$

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   function find ( int a[ ] , int n ) {
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   }
```

$$\tau_5 \equiv \texttt{pc} = 3 \wedge \texttt{c} \geq \texttt{a.length} \wedge \exists x.\ (x \geq 0 \wedge x < \texttt{a.length} \wedge \texttt{a}[x] = \texttt{n}) \wedge \texttt{pc}' = 4$$

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An *array-based system* $\mathcal{S} = (\mathbf{v}, \{\tau_h\})$ is *safe* w.r.t. an error state $U(\mathbf{v})$ iff the formulas

$$I(\mathbf{v}^{(n)}) \wedge \left(\bigvee_h \tau_h(\mathbf{v}^{(n)}, \mathbf{v}^{(n-1)}) \right) \wedge \cdots \wedge \left(\bigvee_h \tau_h(\mathbf{v}^{(1)}, \mathbf{v}^{(0)}) \right) \wedge U(\mathbf{v}^{(0)})$$

are A_I^E-unsatisfiable for every $n \geq 0$

Outline

1. Array-based Transition Systems
2. Unwinding Array-based Transition Systems
3. Refinement with Interpolants
4. Completeness
5. Experiments

Labeled unwinding

A *labeled unwinding* of $\mathcal{S} = \langle \mathbf{v}; \{\tau_h(\mathbf{v}, \mathbf{v}')\}_h \rangle$ is a quadruple (V, E, M_E, M_V)

- (V, E) is a finite rooted tree (let ε be the root)
- M_E, M_V are labeling functions for edges and vertices, respectively

(i) $M_V(\varepsilon) = U(\mathbf{v})$

(ii) $M_V(v)$ is a qff of the kind $\psi(\underline{i}, \mathbf{a}[\underline{i}], \mathbf{c}, \mathbf{d})$ s.t. $M_V(v) \models_{A_I^E} \mathtt{pc} = l$

(iii) $M_E(v, w)$ is the matrix of some $\tau \in \{\tau_h(\mathbf{v}, \mathbf{v}')\}_h$ and

- $M_V(w) \models_{A_I^E} \mathtt{pc} = trg(\tau)$
- $M_V(v) \models_{A_I^E} \mathtt{pc} = src(\tau)$
- $M_E(v, w)(\mathbf{v}, \mathbf{v}') \wedge M_V(w)(\mathbf{v}') \models_{A_I^E} M_V(v)(\mathbf{v})$

(iv) for each $\tau \in \{\tau_h(\mathbf{v}, \mathbf{v}')\}_h$ and every non-leaf vertex $w \in V$ s.t. $M_V(w) \models_{A_I^E} pc = trg(\tau)$, there exist $v \in V$ and $(v, w) \in E$ such that $M_E(v, w)$ is the matrix of τ

Unwinding Array-based Transition Systems

EXPAND procedure

Theorem ([GR10])

If τ is in functional form and $M_V(v)$ is an $\exists^I$-formula[a]*, the pre-image of $M_V(v)$ w.r.t. $\tau(\mathbf{v}, \mathbf{v}')$*

$$Pre(\ M_V(v)\ ,\ \tau\ )\ \ \triangleq\ \ \exists \mathbf{v}'.\ (\ \tau(\mathbf{v}, \mathbf{v}')\ \wedge\ M_V(v)(\mathbf{v}')\ )$$

is A_I^E-equivalent to an effectively computable $\exists^I$-formula

[a] A formula of the kind $\exists \underline{k}.\phi(\underline{k}, \mathbf{a}[\underline{k}], \mathbf{c}, \mathbf{d})$, where $\underline{k}$ have sort `INDEX`

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If τ is in functional form and $M_V(v)$ is an $\exists^I$-formula[a], the pre-image of $M_V(v)$ w.r.t. $\tau(\mathbf{v}, \mathbf{v}')$

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1. Fix the format of the formulas we need to handle
2. It can be shown that the pre-image has all the `INDEX` variables of the starting formula

Unwinding Array-based Transition Systems

Complete unwinding

A label unwinding (V, E, M_V, M_E) is *complete* iff there exists a covering, i.e., a set of non-leaf vertexes C s.t.

- $\varepsilon \in C$
- for every $v \in V$ and $(v', v) \in E$, C covers v', i.e.

$$M_V(v')^{\exists} \models_{A_I^E} \bigvee_{w \in C} M_V(w)^{\exists}$$

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Safe unwinding

A label unwinding (V, E, M_V, M_E) is *safe* iff for all $v \in V$, if $M_V(v) \models_{A_I^E} \mathtt{pc} = l_I$ then $M_V(v)$ is A_I^E-unsatisfiable

- (V, E, M_V, M_E) is not complete
- exists $v \in V$ with a consistent $M_V(v)$, $M_V(v) \models_{A_I^E} \texttt{pc} = l_I$

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Given the path $v = v_0 \xrightarrow{\tau_m} v_1 \xrightarrow{\tau_{m-1}} \cdots \xrightarrow{\tau_1} v_m = \varepsilon$, consider the formula

$$\tau_1(\mathbf{v}^{(0)}, \mathbf{v}^{(1)}) \wedge \cdots \wedge \tau_m(\mathbf{v}^{(m-1)}, \mathbf{v}^{(m)}) \tag{1}$$

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- If (1) is A_I^E-satisfiable, return with UNSAFE
- If (1) is A_I^E-unsatisfiable, retrieve interpolants for $v_0, \ldots, v_m$ to exclude the infeasible counterexample from the model

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Theorem

If T_I- and T_E-satisfiability is decidable, the A_I^E-satisfiability of formulas like

$$\tau_1(\mathbf{v}^{(0)}, \mathbf{v}^{(1)}) \wedge \cdots \wedge \tau_m(\mathbf{v}^{(m-1)}, \mathbf{v}^{(m)})$$

is decidable.

[1] $\mathbf{a}' = \lambda j.G(\ldots)$ is equivalent to $\forall j.\mathbf{a}'[j] = G(\ldots)$.

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Counterexample made by the conjunction of formulas of the kind

$$\exists \underline{k} \begin{pmatrix} \phi(\underline{k}, \mathbf{a}[\underline{k}], \mathbf{c}, \mathbf{d}) \wedge \\ \mathbf{a}' = \lambda j.\, G(\underline{k}, \mathbf{a}[\underline{k}], \mathbf{c}, \mathbf{d}, j, \mathbf{a}[j]) \wedge \\ \mathbf{c}' = H(\underline{k}, \mathbf{a}[\underline{k}], \mathbf{c}, \mathbf{d}) \wedge \mathbf{d}' = K(\underline{k}, \mathbf{a}[\underline{k}], \mathbf{c}, \mathbf{d}) \end{pmatrix}$$

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1. Skolemize $\underline{k}$

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Refinement

Checking satisfiability of the counterexample

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Counterexample made by the conjunction of formulas of the kind

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1. Skolemize $\underline{k}$
2. (Selective) Instantiation of j with Skolem constants[1]

[1] $\mathbf{a}' = \lambda j.G(\ldots)$ is equivalent to $\forall j.\mathbf{a}'[j] = G(\ldots)$.

Refinement

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is decidable.

Counterexample made by the conjunction of formulas of the kind

$$\exists \underline{k} \begin{pmatrix} \phi(\underline{k}, \mathbf{a}[\underline{k}], \mathbf{c}, \mathbf{d}) \wedge \\ \mathbf{a}' = \lambda j.\, G(\underline{k}, \mathbf{a}[\underline{k}], \mathbf{c}, \mathbf{d}, j, \mathbf{a}[j]) \wedge \\ \mathbf{c}' = H(\underline{k}, \mathbf{a}[\underline{k}], \mathbf{c}, \mathbf{d}) \wedge \mathbf{d}' = K(\underline{k}, \mathbf{a}[\underline{k}], \mathbf{c}, \mathbf{d}) \end{pmatrix}$$

1. Skolemize $\underline{k}$
2. (Selective) Instantiation of j with Skolem constants[1]
3. Propagate equalities and exploit decision procedures for T_I and T_E

[1] $\mathbf{a}' = \lambda j.G(\ldots)$ is equivalent to $\forall j.\mathbf{a}'[j] = G(\ldots)$.

Refinement

Retrieving interpolants

Theorem

Given an A_I^E-unsatisfiable quantifier-free formula $\psi_1 \wedge \psi_2$, if

- *T_I and T_E admit quantifier-free interpolation algorithms*
- *all* `INDEX` *variables in ψ_2 under the scope of $\_[\_]$ occur also in ψ_1*

Then, there exists a quantifier-free formula ψ_0 such that:

(i) $\psi_2 \models_{A_I^E} \psi_0$

(ii) *$\psi_0 \wedge \psi_1$ is A_I^E-unsatisfiable*

(iii) *all free variables occurring in ψ_0 occur both in ψ_1 and ψ_2*

```
function find ( int a[ ] , int n ) {
1     c = 0;
2     while (c < a.length ∧ a[c] ≠ n) c = c + 1;
3     if ( c ≥ a.length ∧ ∃x.(x ≥ 0 ∧ x < a.length ∧ a[x] = n) )
4        ERROR;
}
```

$$v_{25} \xrightarrow{\tau_5} v_8 \xrightarrow{\tau_3} v_3 \xrightarrow{\tau_2} v_1 \xrightarrow{\tau_1} \epsilon$$

$$\{\mathtt{pc} = 1\} \qquad \{\mathtt{pc} = 2\} \qquad \left\{ \begin{array}{l} \mathtt{pc} = 2 \wedge \\ \mathtt{c} > 0 \wedge \\ \mathtt{a}[i_0] = \mathtt{n} \end{array} \right\} \qquad \left\{ \begin{array}{l} \mathtt{pc} = 3 \wedge \\ \mathtt{c} > 0 \wedge \mathtt{a}[i_0] = \mathtt{n} \wedge \\ i_0 < \mathtt{c} \wedge \mathtt{a.length} \geq 1 \end{array} \right\} \qquad \{\mathtt{pc} = 4\}$$

```
function find ( int a[ ] , int n ) {
    c = 0;
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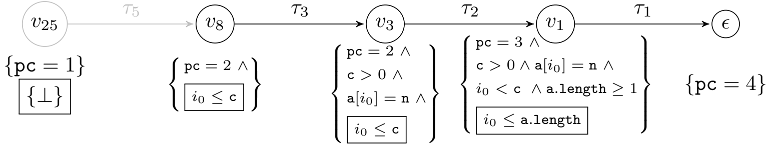

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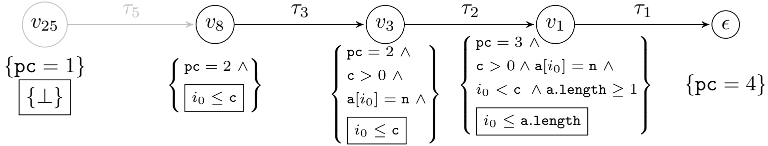

$$(\mathtt{pc} = 3 \wedge \mathtt{c} > 0 \wedge \mathtt{a.length} \geq 1) \Rightarrow \forall i.\ ((i < \mathtt{c} \wedge i \leq \mathtt{a.length}) \Rightarrow \mathtt{a}[i] \neq \mathtt{n})$$

Outline

1 Array-based Transition Systems

2 Unwinding Array-based Transition Systems

3 Refinement with Interpolants

4 Completeness

5 Experiments

Completeness

Covering set C: for every $v \in V$ and $(v', v) \in E$,

$$M_V(v')^{\exists} \models_{A_I^E} \bigvee_{w \in C} M_V(w)^{\exists}$$

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Theorem ([GR10])

If Σ_I does not contain function symbols and $\mathcal{C}_I$ is closed under substructures, the A_I^E-satisfiability of formulas of the form

$$\exists \mathbf{a}\ \exists \mathbf{c}\ \exists \mathbf{d}\ \exists \underline{i}\ \forall \underline{j}\ \ \psi(\underline{i}, \underline{j}, \mathbf{a}[\underline{i}], \mathbf{a}[\underline{j}], \mathbf{c}, \mathbf{d})$$

is decidable, where ψ is a quantifier-free $\Sigma_I \cup \Sigma_E$-formula.

Completeness (and Termination)
Discussion

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- What's the behavior of UNWIND in practice (even on systems not meeting termination hypothesis)?

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1. Array-based Transition Systems
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4. Completeness
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Experiments

- SAFARI (SMT-based Abstraction For Arrays with Interpolants) - `http://www.verify.inf.usi.ch/safari`
  - Integration with OpenSMT for SMT-solving

- Experiments over imperative programs handling arrays

Benchmark	Time (s)	Nodes	SMT-calls	Iter.	Result
find (v1)	0.3	5	192	3	SAFE
find (v2)	0.07	5	48	1	SAFE
initialization	0.1	5	96	1	SAFE
max in array	0.9	72	1192	8	SAFE
partition	0.08	20	62	0	SAFE
strcmp	0.4	14	329	4	SAFE
strcpy	0.03	3	15	0	SAFE
vararg	0.03	5	17	0	SAFE
integers	0.02	5	19	0	SAFE
init and test	0.3	27	375	3	SAFE
binary sort	0.3	48	457	2	SAFE
selection sort	0.6	15	478	4	SAFE

Intel i7 @2.66 GHz, equipped with 4GB of RAM and running OSX 10.7

More examples on `http://verify.inf.usi.ch/safari`

Related work

- Ghost variables [FQ02]
- Index predicates [LB07]
- Range predicates [JM07]
- Theorem prover based [KV09, HKV11]
- Abstract interpretation [CCL11, HP08, DDA10]

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- Future work
  - New powerful heuristics for "tuning" interpolation and help convergence (currently under submission)
  - Use SAFARI as invariant generator
  - Other classes of systems (e.g., distributed algorithms)

Thank you!
Questions?

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