Acceleration-based Safety Decision Procedure for Programs with Arrays

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LPAR-19

December 15, 2013

Talk based on results previously published at FroCoS 2013.

Context: decide the safety of programs with arrays

$$
\begin{array}{ll}
 & \textsf{procedure Find}(\ \mathtt{a}[\mathtt{L}]\ ,\ \mathtt{e}\ )\ \{ \\
l_I & \quad \mathtt{i} = 0; \\
l_L & \quad \textsf{while}\ (\ \mathtt{i} < \mathtt{L} \land \mathtt{a}[\mathtt{i}] \neq \mathtt{e}\ )\ \{ \\
 & \qquad \mathtt{i} = \mathtt{i} + 1; \\
 & \quad \} \\
l_F & \quad \textsf{assert}\ (\ \forall x.(0 \leq x < \mathtt{i}) \rightarrow \mathtt{a}[x] \neq \mathtt{e}\ ); \\
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\end{array}
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- Is this program safe?
- Can we *decide* its safety automatically?

Our solution

Problem:

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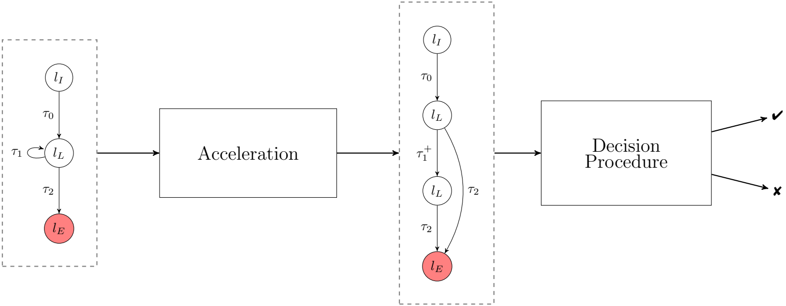
Idea:



Formal framework

$$\mathcal{S}_T \;=\; (\; \mathbf{v} \;,\; I(\mathbf{v}) \;,\; \tau(\mathbf{v}, \mathbf{v}') \;)$$

[1] In all the formulæ we admit the term $a(t)$ only if t is a variable or a constant.

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$$\mathcal{S}_T \;=\; (\; \mathbf{v} \;,\; I(\mathbf{v}) \;,\; \tau(\mathbf{v}, \mathbf{v}') \;)$$

- T is Presburger arithmetic enriched with free function symbols
  - satisfiability and validity with respect to structures having the standard structure of natural numbers as reduct
  - $\mathbf{v}$ contains free unary function symbols ($\mathbf{a}$) and free constants ($\mathbf{c}$)

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- Σ_2^0 – formulas of the kind $\exists \underline{i} \forall \underline{j}.\phi(\underline{i},\underline{j},\mathbf{v})$

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State of the art

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- Polling-based systems [BBD$^+$02]
- Imperative programs over integers [BIK10]

- What about arrays?
  - Acceleration of *local ground assignment* [AGS13] can be expressed in the theory T as Σ_2^0-assignments

Acceleration for arrays
Example

$$\tau_1 := pc = l_L \quad \wedge \quad \underbrace{\mathtt{i} < \mathtt{L} \wedge \mathtt{a}[\mathtt{i}] \neq \mathtt{e}}_{\text{guard}} \quad \wedge \quad \underbrace{\mathtt{i}' = \mathtt{i} + 1}_{\text{update}}$$

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$$\tau_1^+ := \exists y. \begin{pmatrix} y > 0 \wedge pc = l_L \ \wedge \\ \forall j.(\ \mathtt{i} \leq j < \mathtt{i} + y \quad \rightarrow \quad j < \mathtt{L} \wedge \mathtt{a}[j] \neq \mathtt{e}\ ) \wedge \\ \mathtt{i}' = \mathtt{i} + y \end{pmatrix}$$

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Do the "jump"

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II. Notion of *basic-flat-programs*

- *flat* control flow graph
- every non-loop edge is labeled with a ground or Σ_1^0-assignment
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II. Notion of *basic-flat-programs*
- *flat* control flow graph
- every non-loop edge is labeled with a ground or Σ_1^0-assignment
- every loop edge is labeled with a *basic-assignment*.

III. The reachability problem for *basic-flat-programs* is **decidable**
1. Accelerate all the loops (basic-assignments)
2. Consider all (finitely many) paths from l_{init} to l_{error}

   $\Rightarrow$ Feasible iff the corresponding *Array Property formula* is satisfiable

Basic-Flat-Programs

Procedures handling arrays of unknown length like:

- Initialization of the array to a given value
- Searching in an array for a given value
- Swapping two different arrays
- Testing if two arrays are equal

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Future work: new decidability results for array programs based on

- New decidable (quantified) fragments of array theories
- New acceleration schemata for assignments modeling pieces of code

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Thank you! Questions?

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